Self-Fulfilling Credit Cycles*

Costas Azariadis† Leo Kaas‡

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Abstract

We develop a tractable dynamic equilibrium model of cyclical capital misallocation arising from limited credit enforcement. Heterogeneous firms are faced with idiosyncratic productivity shocks. Capital from less productive firms is lent to more productive ones in the form of credit secured by collateral, and also as unsecured credit based on reputation. A dynamic complementarity between current and future endogenous credit limits permits uncorrelated sunspot shocks to trigger persistent fluctuations in debt, aggregate productivity and output. For reasonably calibrated parameter values we demonstrate the existence of endogenous cycles featuring recurrent credit booms and credit slumps and plausible dynamics for output, firm credit and investment.

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†Department of Economics, Washington University, St. Louis MO 63130-4899, USA, and Federal Reserve Bank of St.Louis E-mail: azariadi@artsci.wustl.edu

‡Department of Economics, University of Konstanz, Box D145, 78457 Konstanz, Germany. E-mail: leo.kaas@uni-konstanz.de

1 Introduction

Over the past two decades, there have been important advances in macroeconomic research demonstrating that financial market conditions play a key role for business cycle fluctuations. Starting with seminal contributions of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), much of this research shows how frictions in financial markets can amplify and propagate disruptions to macroeconomic fundamentals, such as shocks to total factor productivity (TFP) or to monetary policy.[1] More recently, and to some extent motivated by the events of the recent financial crisis, several theoretical and quantitative contributions argue that shocks to the financial sector itself may not only lead to severe macroeconomic consequences but can also contribute significantly to business cycle movements. For example, Jermann and Quadrini (2012) develop a model with stochastic collateral constraints which they identify as residuals from aggregate time series of firm debt and collateral (capital). Estimating a joint stochastic process for TFP and borrowing constraints, they find that both variables are highly autocorrelated and that financial shocks play an important role for business cycle fluctuations.[2] But what drives these shocks to borrowing constraints and to aggregate productivity? And what makes their responses highly persistent?

This paper proposes a common underlying source for these observations: endogenous volatility in the form of self-fulfilling expectations of credit market conditions. We develop and analyze a parsimonious dynamic general equilibrium model with heterogeneous firms and limited enforcement of unsecured credit. In the model, borrowing constraints and TFP are endogenous variables. Constraints on unsecured credit depend on the value that borrowers attach to future credit market conditions which is a forward-looking variable. TFP depends on the capital allocation between heterogeneous firms which, among others, depends on current credit constraints. When these constraints bind, they slow down capital reallocation between firms and push aggregate factor productivity below its frontier. We show that this model exhibits a very natural equilibrium indeterminacy which gives rise to endogenous cycles driven by self-fulfilling beliefs in credit market conditions (sunspot shocks). In particu-

[1] For recent surveys, see Quadrini (2011) and Brunnermeier, Eisenbach, and Sannikov (2012).

[2] Other examples of financial shocks are Kiyotaki and Moore (2008) who introduce shocks to asset resaleability and Gertler and Karadi (2011) who consider shocks to the asset quality of financial intermediaries. Both also assume highly persistent shock processes in their simulation experiments.

lar, a one-time sunspot shock triggers an endogenous and persistent response of endogenous borrowing constraints and of TFP.

The model is a standard stochastic growth model which comprises a large number of firms facing idiosyncratic productivity shocks. In each period, productive firms wish to borrow from their less productive counterparts. These firms exchange secured and unsecured credit. Secured credit is restricted by the firm's collateral which is determined by an exogenous fraction of the firm's wealth, similar to Kiyotaki and Moore (1997). Unsecured credit rests on the borrower's reputation. Building upon Bulow and Rogoff (1989) and Kehoe and Levine (1993), we assume that a defaulting borrower is excluded from future credit for a stochastic number of periods. As in Alvarez and Jermann (2000), endogenous forward–looking credit limits prevent default. These credit limits depend on the value that a borrower attaches to a good reputation which itself depends on future credit market conditions.

An important contribution of this paper is the tractability of our framework which permits us to derive a number of insightful analytical results in Sections 3 and 4. With standard and convenient specifications of preferences and technology, we characterize any equilibrium by one backward-looking and one forward-looking equation (Proposition 1). With this characterization, we prove that unsecured credit cannot support first-best allocations, unless collateral constraints are sufficiently loose, thereby extending related findings of Bulow and Rogoff (1989) and Hellwig and Lorenzoni (2009) to a growth model with idiosyncratic productivity (Proposition 2). We then prove the existence of multiple stationary equilibria for a range of parameter configurations (Proposition 3). While an equilibrium without unsecured credit always exists, there can also exist one or two stationary equilibria with a positive volume of unsecured credit. One of these equilibria has an efficient allocation of capital between firms, and another one features a misallocation of capital. The latter equilibrium is the one that provides the most interesting insights, since unsecured credit is traded and yet factor productivity falls short of the efficient technology frontier.[3] We show that this equilibrium is always locally indeterminate, and hence that it permits the existence of sunspot cycles fluctuating around the stationary equilibrium (Proposition 4). Moreover, output and credit respond persistently to a

[3]The other, determinate steady states of this model either do not sustain unsecured credit (and hence resemble similar dynamics as in a Kiyotaki–More–type model with binding collateral constraints) or they have an efficient allocation of capital (and hence exhibit the same business cycle properties as a frictionless model).

one-time sunspot shock.

In Section 5 we calibrate our model to the U.S. economy and show that an *uncorrelated* sunspot process generates *persistent* and plausible dynamics of key macroeconomic variables. Particularly, the model captures the relative volatilities and autocorrelation patterns of output, firm credit and investment reasonably well. Such adjustment dynamics cannot be generated by any of the typical fundamental shocks once the underlying shock process is uncorrelated. Although this fact is well known, we still contrast the resulting dynamics by feeding our model with uncorrelated shocks to the collateral share (financial shocks), to the productivity spread between firms (uncertainty shocks) or to the share of investing firms (investment spikes). Neither of these processes can generate a persistent macroeconomic response. On the other hand, a misspecified model would easily identify fundamental shock processes as highly persistent if data were generated by our model with uncorrelated and non-fundamental shocks.

Intuitively, the explanation for indeterminacy and sunspot cycles is a dynamic complementarity in endogenous constraints on unsecured credit. Borrowers' incentives to default depend on their expectations of future credit market conditions, which in turn takes an impact on current credit constraints. If borrowers expect a credit tightening over the next few periods, their current default incentives become larger which triggers a tightening of current credit. This insight also explains why a one-time expectation shock *must be followed* by a long-lasting response of credit market conditions (and thus of macroeconomic outcomes): if market participants would expect that a credit boom (or a credit slump) dies out quickly, such expectations could not be powerful enough to generate a sizable credit boom (or slump) today.

Another way to understand the role of expectations is that unsecured credit shares similarities with a bubble which is sustained by self-fulfilling beliefs, as has been argued by Hellwig and Lorenzoni (2009). Transitions from a "good" macroeconomic outcome with plenty of unsecured credit to a "bad" outcome with low volumes of unsecured credit can be triggered by widespread skepticism about the ability of financial markets to continue the provision of unsecured credit at the volume needed to support socially desirable outcomes, which is similar to the collapse of a speculative bubble. The emergence and the bursting of rational bubbles in financially constrained economies has received attention in a number of recent contributions, e.g. Caballero and Krishnamurthy (2006), Kocherlakota (2009), Farhi and Tirole (2011) and Miao and Wang (2012). One difficulty with many of the existing

macroeconomic models with bubbles is that the no-bubble equilibrium is an attracting steady state, so that they can only account for the bursting of bubbles but not for their building up.[4] Although there are no bubbles in our model, its equilibrium dynamics account for recurrent episodes of credit booms and busts which are solely driven by self-fulfilling beliefs.

Our work is also related to a literature on sunspot cycles arising from financial frictions. In an early contribution, Woodford (1986) shows that a simple borrowing constraint makes infinitely-lived agents behave like two-period-lived overlapping generations, so that endogenous cycles can occur with sufficiently strong income effects or with increasing returns in production (see e.g. Benhabib and Farmer (1999) for a survey). Harrison and Weder (2010) introduce a production externality in a Kiyotaki-Moore (1997) model and show that sunspots emerge for reasonable values of returns to scale. Other recent contributions find equilibrium indeterminacy and multiplicity in endowment economies with limited credit enforcement under specific assumptions about trading arrangements (Gu and Wright (2011)) and on the enforcement technology (Azariadis and Kaas (2011)).[5] Perri and Quadrini (2011) develop a two-country model with financial frictions and show that self-fulfilling expectations of asset values may be responsible for the international synchronization of credit tightening.

The rest of this paper is organized as follows. Section 2 lays out the model framework and defines competitive equilibrium. In Section 3 we characterize all equilibria by a forward-looking equation in the reputation values of borrowers and we show that unsecured credit cannot overcome binding constraints, unless the collateral share is sufficiently large. Section 4 derives our main results on equilibrium multiplicity, indeterminacy and sunspot cycles. In Section 5 we consider a calibrated numerical example to highlight the different impacts of sunspot shocks and fundamental shocks for business cycle dynamics. Section 6 concludes. The Appendix contains an extension to a model with autocorrelated firm productivity and all proofs for those results which are not obvious from the main text.

[4] A recent exception is Martin and Ventura (2011) who construct an overlapping-generations model with two types of investors and permanent stochastic bubble dynamics.

[5] Azariadis and Kaas (2012) consider a multi-sector endogenous growth model under limited commitment and also find equilibrium multiplicity.

2 The model

Consider a growth model in discrete time with a continuum $i \in [0,1]$ of firms, each owned by a single entrepreneur, and a unit mass of workers. At any date t, all agents maximize expected discounted utility

$$E_t(1-\beta)\sum_{\tau \geq t}\beta^{\tau-t}\log(c_\tau) \tag{1}$$

over future consumption streams. Workers supply one unit of labor per period and have no capital endowment. Entrepreneurs own capital and have no labor endowment. They produce a consumption and investment good y_t using capital k'_t and labor ℓ_t with common constant-returns technology $y_t = (k'_t)^\alpha (A_t \ell_t)^{1-\alpha}$. Aggregate labor efficiency A_t grows at rate g.

Entrepreneurs differ in their ability to operate capital investment k_t. Some entrepreneurs are able to enhance their invested capital according to $k'_t = a^p k_t$; these entrepreneurs are labeled "productive". The remaining, "unproductive" entrepreneurs deplete some of their capital investment such that $k'_t = a^u k_t$. We assume that $a^p > 1 > a^u$ and write $\gamma \equiv a^u/a^p$ for the relative productivity gap. To the next period, all capital depreciates at common rate δ. Productivity realizations are independent across agents and uncorrelated across time;[6] entrepreneurs are productive with probability π and unproductive with probability $1-\pi$. Thus, fraction π of the aggregate capital stock K_t is owned by productive entrepreneurs in any period.

Timing within each period is as follows. *First* entrepreneurs learn their productivity, they borrow and lend in a centralized credit market at gross interest rate R_t, and they hire labor in a centralized labor market at wage w_t. *Second* production takes place. *Third*, entrepreneurs redeem their debt; agents consume and save for the next period.

In the credit market, productive entrepreneurs borrow capital from unproductive entrepreneurs. They are able to pledge an exogenous fraction $\lambda < 1$ of their total wealth (output and undepreciated capital) as collateral which can be seized in the event of default. All borrowers have access to such secured credit. On top of that, agents can borrow unsecured if they have a clean credit record. However, if a

[6] In Appendix A we characterize solutions of this model with correlated productivity shocks. The complication is that the wealth distribution enters as an additional state variable, but our main findings on equilibrium multiplicity remain intact.

borrower decides to default in some period, his credit record deteriorates and the entrepreneur is banned from unsecured credit for a stochastic number of periods. Defaulters are still allowed to lend, however, and they can also pledge assets to creditors so that they have access to secured credit. Each period after default, the entrepreneur's credit record is cleared with probability ψ in which case the entrepreneur regains full access to credit markets.

Since no shocks arrive during a credit contract (that is, debt is redeemed at the end of the period before the next productivity shock is realized), there exist default–deterring credit limits, defined similarly as in the pure–exchange model of Alvarez and Jermann (2000). These limits are the highest values of credit that prevent default. In the absence of secured credit ($\lambda = 0$), this enforcement technology corresponds to the one discussed by Bulow and Rogoff (1989) and Hellwig and Lorenzoni (2009) who consider unsecured credit and assume that defaulters are excluded from future credit but are still allowed to save. With $\lambda > 0$, secured credit is available and sometimes, but not always, a higher flow of credit can be sustained. Unsecured borrowing is founded on a producer's desire to maintain a clean credit record and hence continued access to future unsecured credit. The value of λ is constant and common for all entrepreneurs;[7] it depends on technological factors like the collaterizability of income and wealth, as well as on creditor rights and other aspects of economic institutions. We assume that $\lambda < \gamma(1-\pi)$; this restriction implies that credit constraints are binding on borrowers in equilibrium (see Proposition 2).

While entrepreneurs can borrow in the credit market, we assume that workers' labor income cannot be collateralized and that they have no access to unsecured credit either. That is, workers face a zero borrowing constraint. Further, as we show below, $R < (1+g)/\beta$ holds in any stationary equilibrium, and hence also in any cyclical equilibrium near the steady state. Thus, workers are borrowing constrained and do not want to save; they simply consume their wage income in every period.

Let θ_t denote the endogenous constraint on a borrower's debt–equity ratio in period t, the same for all borrowers with a clean credit record. If a productive entrepreneur enters the period with equity e_t, he borrows $b_t = \theta_t e_t$ and invests $k_t = e_t + b_t$. An unproductive entrepreneur lends out capital, so $b_t \leq 0$, and investment is $k_t =$

[7] It is only a matter of additional notation to augment this model by a stochastic process for λ_t or for any other fundamental parameter. We do this in Section 5 to illustrate the effects of fundamental shocks.

$e_t + b_t \leq e_t$. The budget constraint for an entrepreneur with capital productivity $a^s \in \{a^p, a^u\}$ reads as

$$c_t + e_{t+1} = (a^s k_t)^\alpha (A_t \ell_t)^{1-\alpha} + (1-\delta) a^s k_t - w_t \ell_t - R_t b_t \ . \tag{2}$$

We are now ready to define equilibrium.

Definition: *A competitive equilibrium is a list of consumption, savings, and production plans for all entrepreneurs,* $(c_t^i, e_t^i, b_t^i, k_t^i, \ell_t^i)_{i\in[0,1]}$*, contingent on realizations of idiosyncratic productivities, consumption of workers,* $c_t^w = w_t$*, factor prices for labor and capital* (w_t, R_t)*, and debt-equity constraints* θ_t*, such that in every period* $t \geq 0$*:*[8]

(i) $(c_t^i, e_t^i, b_t^i, k_t^i, \ell_t^i)$ *maximizes entrepreneur* i*'s expected discounted utility (1) subject to budget constraints (2) and credit constraints* $b_t^i \leq \theta_t e_t^i$*.*

(ii) *Markets for labor and capital clear;*

$$\begin{aligned} \int_0^1 \ell_t^i \, di &= 1 \ , \\ \int_0^1 b_t^i \, di &= 0 \ . \end{aligned}$$

(iii) *If* $b_t^i \leq \theta e_t^i$ *is binding in problem (i), entrepreneur* i *is exactly indifferent between debt redemption and default in period* t*, where default entails the loss of collateral and the exclusion from unsecured credit for a geometrically distributed number of periods with mean* $1/\psi$*.*

3 Equilibrium characterization

Since entrepreneurs hire labor so as to equate the marginal product to the real wage, all productive (unproductive) entrepreneurs have identical capital–labor ratios; they are linked according to

$$\frac{k_t^p}{\ell_t^p} = \gamma \frac{k_t^u}{\ell_t^u} \ . \tag{3}$$

[8] In period $t = 0$, there is some given initial equity distribution $(e_0^i)_{i\in[0,1]}$.

With binding credit constraints, fraction $z_t \equiv \min[1, \pi(1+\theta_t)]$ of the aggregate capital stock K_t is operated by productive entrepreneurs. It follows from (3) and labor market clearing that

$$\frac{k_t^p}{\ell_t^p} = \frac{a_t K_t}{a^p} \leq K_t < \frac{a_t K_t}{a^u} = \frac{k_t^u}{\ell_t^u} \ ,$$

where $a_t \equiv a^p z_t + a^u(1-z_t)$ is the average capital productivity. The gross return on capital for an entrepreneur with capital productivity $a^s \in \{a^u, a^p\}$ is then $a^s R_t^*$, with capital return $R_t^* \equiv 1 - \delta + \alpha A_t^{1-\alpha}(a_t K_t)^{\alpha-1}$.

In any equilibrium, the gross interest rate cannot exceed the capital return of productive entrepreneurs $a^p R_t^*$. It also cannot fall below the capital return of unproductive entrepreneurs $a^u R_t^*$. Thus it is convenient to write $R_t = \rho_t a^p R_t^*$ with $\rho_t \in [\gamma, 1]$. When $\rho_t < 1$, borrowers are credit constrained. In this case the leveraged equity return

$$\tilde{R}_t \equiv [1 + \theta_t(1-\rho_t)] a^p R_t^* \tag{4}$$

exceeds the capital return $a^p R_t^*$. Unproductive entrepreneurs, on the other hand, lend out all their capital when $\rho_t > \gamma$; they only invest in their own inferior technology if $\rho_t = \gamma$. Therefore, credit market equilibrium amounts to the complementary-slackness condition

$$\rho_t \geq \gamma \quad , \quad \pi(1+\theta_t) \leq 1 \ . \tag{5}$$

With this notation, the entrepreneurs' budget constraints (2) simplify to $e_{t+1} + c_t = \tilde{R}_t e_t$, when the entrepreneur is productive in t, and to $e_{t+1} + c_t = R_t e_t$ when the entrepreneur is unproductive. From logarithmic utility follows that every entrepreneur consumes fraction $(1-\beta)$ and saves fraction β of wealth.

To derive the endogenous credit limits, let $V_t(W)$ denote the continuation value of an entrepreneur with a clean credit record who has wealth W at the end of period t, prior to deciding consumption and saving. These values satisfy the recursive equation[9]

$$V_t(W) = (1-\beta)\log[(1-\beta)W] + \beta\pi E_t V_{t+1}(\tilde{R}_{t+1}\beta W) + \beta(1-\pi)E_t V_{t+1}(R_{t+1}\beta W) \ .$$

The first term in this equation represents utility from consumption $(1-\beta)W$ in the current period. To the next period $t+1$, the entrepreneur saves equity βW which

[9] In the absence of aggregate risk (sunspot or fundamental shocks), the expectations operator E_t could be dropped from this and from subsequent recursive equations.

earns return $\tilde{R}_{t+1}$ with probability π and return R_{t+1} with probability $1-\pi$. It follows that continuation values take the form $V_t(W) = \log(W) + V_t$ where V_t is independent of wealth, satisfying

$$V_t = (1-\beta)\log(1-\beta) + \beta\log\beta + \beta E_t\left[\pi\log\tilde{R}_{t+1} + (1-\pi)\log R_{t+1} + V_{t+1}\right] . \quad (6)$$

If an entrepreneur has a bad credit record, he is only permitted to borrow against collateral. His debt-equity ratio θ_t^c then ensures that the value of debt does not exceed the value of collateral. Hence it is pinned down from $R_t\theta_t^c = \lambda a^p R_t^*(1+\theta_t^c)$ which yields $\theta_t^c = \lambda/(\rho_t - \lambda)$ and equity return

$$\tilde{R}_t^c \equiv [1+\theta_t^c(1-\rho_t)]a^p R_t^* = \frac{\rho_t(1-\lambda)}{\rho_t-\lambda}a^p R_t^* . \quad (7)$$

We write the continuation value of such an entrepreneur $V_t^c(W) = \log(W) + V_t^c$, where V_t^c satisfies, analogously to equation (6),

$$\begin{aligned} V_t^c &= (1-\beta)\log(1-\beta) + \beta\log\beta \quad (8)\\ &\quad +\beta E_t\left[\pi\log\tilde{R}_{t+1}^c + (1-\pi)\log R_{t+1} + V_{t+1}^c + \psi(V_{t+1} - V_{t+1}^c)\right] . \end{aligned}$$

This entrepreneur is banned from unsecured credit in period $t+1$ so that the equity return is $\tilde{R}_{t+1}^c$ with probability π and R_{t+1} with probability $1-\pi$. At the end of period $t+1$, the entrepreneur's credit record clears with probability ψ in which case continuation utility increases from V_{t+1}^c to V_{t+1}.

If a productive entrepreneur has a clean credit record and enters period t with equity e_t, the debt-equity constraint θ_t makes him exactly indifferent between default and debt redemption if

$$\log\left[\tilde{R}_t e_t\right] + V_t = \log\left[(1-\lambda)a^p R_t^*(1+\theta_t)e_t\right] + V_t^c .$$

Here the right-hand side is the continuation value after default: the entrepreneur invests $(1+\theta_t)e_t$, earns return $a^p R_t^*$ and retains the uncollateralized share of wealth. The left-hand side is the continuation value under solvency, where the entrepreneur earns equity return $\tilde{R}_t$. Defining $v_t \equiv V_t - V_t^c \geq 0$ as the "value of reputation", this equation can be solved for the default-deterring credit constraint

$$\theta_t = \frac{e^{v_t} - 1 + \lambda}{1 - \lambda - e^{v_t}(1-\rho_t)} . \quad (9)$$

This solution has a rather insightful interpretation. First, the debt-equity ratio is increasing in the reputation value v_t: a greater expected payoff from access to

unsecured credit makes debt redemption more valuable, which relaxes the credit constraint. In fact, when the reputation value is zero, unsecured credit cannot be sustained and all credit is secured, so that $\theta_t = \theta_t^c$. Second, and quite obviously, the debt-equity ratio is increasing in the collateral share λ.

Using (4), (6), (7) and (8), reputation values satisfy the recursive identity

$$\begin{aligned} v_t &= \beta E_t\Big[\pi \log \frac{\tilde{R}_{t+1}}{\tilde{R}^c_{t+1}} + (1-\psi)v_{t+1}\Big] \\ &= \beta E_t\Big[\pi \log\Big(\frac{\rho_{t+1}-\lambda}{1-\lambda-e^{v_{t+1}}(1-\rho_{t+1})}\Big) + (1-\psi)v_{t+1}\Big] \ . \end{aligned} \tag{10}$$

We can summarize this equilibrium characterization as follows.

Proposition 1 *Any solution $(\rho_t, \theta_t, v_t)_{t\geq 0}$ to the system of equations (5), (9) and (10) gives rise to a competitive equilibrium with interest rates $R_t = \rho_t a^p R_t^*$ and capital returns $R_t^* = 1-\delta+\alpha A_t^{1-\alpha}(a_t K_t)^{\alpha-1}$ with $a_t = a^u + (a^p - a^u)\cdot\min[1, \pi(1+\theta_t)]$. The aggregate capital stock evolves according to*

$$K_{t+1} = \beta\Big[(1-\delta) + \alpha A_t^{1-\alpha}(a_t K_t)^{\alpha-1}\Big] a_t K_t \ . \tag{11}$$

An implication of this proposition is that any equilibrium follows two dynamic equations, the backward-looking dynamics of aggregate capital (equation (11)) and the forward-looking dynamics of reputation values, see equation (12) below. Due to our modeling of the idiosyncratic productivity process, the latter identity is even independent of the aggregate state K_t, and hence permits a particularly simple analysis of stationary and non-stationary equilibria.[10]

Using Proposition 1, we obtain two immediate results. First, there always exists an equilibrium where unsecured credit is not available. Formally, $v_t = 0$, $\theta_t = \theta_t^c = \lambda/(\gamma-\lambda)$ and $\rho_t = \gamma$ solves the above equilibrium conditions, given our parameter restriction on collateral. Intuitively, if no unsecured credit is available in the future, there is no value of reputation, and hence any borrower prefers to default on current unsecured credit. It follows that no unsecured credit is available in the current

[10] On the one hand, reputation values are independent of aggregate capital since excess returns are multiples of the capital return R_t^* which follows from our assumption that firms differ in capital productivity, $k_t' = a^s k_t$ for $s = u, p$. If firms had different total factor productivity instead, equation (12) would depend on K_t. On the other hand, if productivity shocks were autocorrelated, the wealth distribution would enter as a state variable into equation (12), see Appendix A.

period.[11] Second, we can show that unsecured credit cannot overcome binding borrowing constraints. This is in line with earlier results by Bulow and Rogoff (1989) and Hellwig and Lorenzoni (2009) who show that the first best cannot be implemented by limited enforcement mechanisms which ban defaulting agents from future borrowing but not from future lending. It differs decisively from environments with two–sided exclusion, as in Kehoe and Levine (1993) and Alvarez and Jermann (2000), where first-best allocations can be sustained with unsecured credit under certain circumstances.[12] The intuition for this result is as follows. If borrowers were unconstrained, the interest rate would coincide with the borrowers' capital return. Hence there is no leverage gain, so that access to credit has no value for borrowers. In turn, every borrower would default on an unsecured loan, no matter how small. We summarize this finding in

Proposition 2 *Suppose that $\lambda < \gamma(1-\pi)$. Then any equilibrium features binding borrowing constraints.*

It follows immediately that the equilibrium interest rate is smaller than the workers' marginal rate of intertemporal substitution, so that workers are indeed credit constrained.

Corollary 1 *In any steady state equilibrium (balanced growth path), $R < \frac{1+g}{\beta}$.*

4 Multiplicity and cycles

Although borrowers must be credit constrained in any equilibrium, the credit market may nonetheless be able to allocate capital efficiently. In particular, when the reputation value v_t is sufficiently large, credit constraints relax and the interest rate exceeds the capital return of unproductive entrepreneurs who lend out all their capital to more productive entrepreneurs. Formally, when v_t exceeds the threshold value

$$\overline{v} \equiv \log\left[\frac{1-\lambda}{1-\gamma(1-\pi)}\right] > 0 \ ,$$

[11] A similar "autarky" equilibrium also obtains in the limited enforcement economy of Kehoe and Levine (1993) as decentralized by Alvarez and Jermann (2000); cf. Azariadis and Kaas (2007).

[12] In endowment economies it is well known that perfect risk sharing can be implemented if the discount factor is sufficiently large, if risk aversion is sufficiently strong or if the endowment gap between agents is large enough (see e.g. Kehoe and Levine (2001) and Azariadis and Kaas (2007)).

the equilibrium conditions (5) and (9) are solved by $\theta_t = (1-\pi)/\pi$ and $\rho_t = [1-e^{-v_t}(1-\lambda)]/(1-\pi) > \gamma$. Conversely, when v_t falls short of $\overline{v}$, credit constraints tighten, the interest rate equals the capital return of unproductive entrepreneurs ($\rho_t = \gamma$), who are then indifferent between lending out capital or investing in their own technology, so that some capital is inefficiently allocated. We can use this insight to rewrite the forward-looking equation (10) as

$$v_t = E_t f(v_{t+1}) \ , \tag{12}$$

with

$$f(v) \equiv \begin{cases} \beta(1-\psi)v + \beta\pi \log\left[\dfrac{\gamma-\lambda}{1-\lambda-e^v(1-\gamma)}\right] & , \ \text{if } v \in [0,\overline{v}] \ , \\ \beta(1-\pi-\psi)v + \beta\pi \log\left[\dfrac{e^v[1-\lambda(1-\pi)]+\lambda-1}{\pi(e^v+\lambda-1)}\right] & , \ \text{if } v \in [\overline{v}, v_{max}] \ . \end{cases}$$

Here $v = v_{max} = \log(\frac{1-\lambda}{\pi})$ is the reputation value where the interest rate satisfies $\rho = 1$ and borrowers are unconstrained. It is straightforward to verify that f is strictly increasing if $\pi + \psi < 1$, convex in $v < \overline{v}$ and concave in $v > \overline{v}$, and it satisfies $f(0) = 0$ and $f(v_{max}) < v_{max}$. This confirms again that the absence of unsecured credit ($v = 0$) is a stationary equilibrium of this model. Depending on economic fundamentals, there can also exist one or two steady states exhibiting positive trading of unsecured credit. Figure 1(a) shows a situation in which function f has three intersections with the 45-degree line: $v = 0$, $v^* \in (0, \overline{v})$ and $v^{**} \in (\overline{v}, v_{max})$. The steady states at $v = 0$ and at v^* have an inefficient capital allocation. At $v^{**} > \overline{v}$, capital is efficiently allocated. Figure 1(b) shows a possibility with only two steady states, at $v = 0$ and at $v^{**} > \overline{v}$. The following proposition describes how the set of stationary equilibria changes as the productivity gap $\gamma = a^u/a^p$ varies.

Proposition 3 *There exists a stationary equilibrium in which no unsecured credit is available. Provided that* $\lambda < \dfrac{\beta(1-\pi)}{1+\beta\psi}$, *there are threshold values* $\gamma_0 < \gamma_1 < 1$ *such that:*

(a) *For* $\gamma \in (\gamma_0, \gamma_1)$, *there are two stationary equilibria with unsecured credit: one at* $v^* \in (0, \overline{v})$ *with inefficient capital allocation and one at* $v^{**} \in (\overline{v}, v_{max})$ *with efficient capital allocation.*

(b) *For* $\gamma > \gamma_1$, *there is no stationary equilibrium with unsecured credit.*

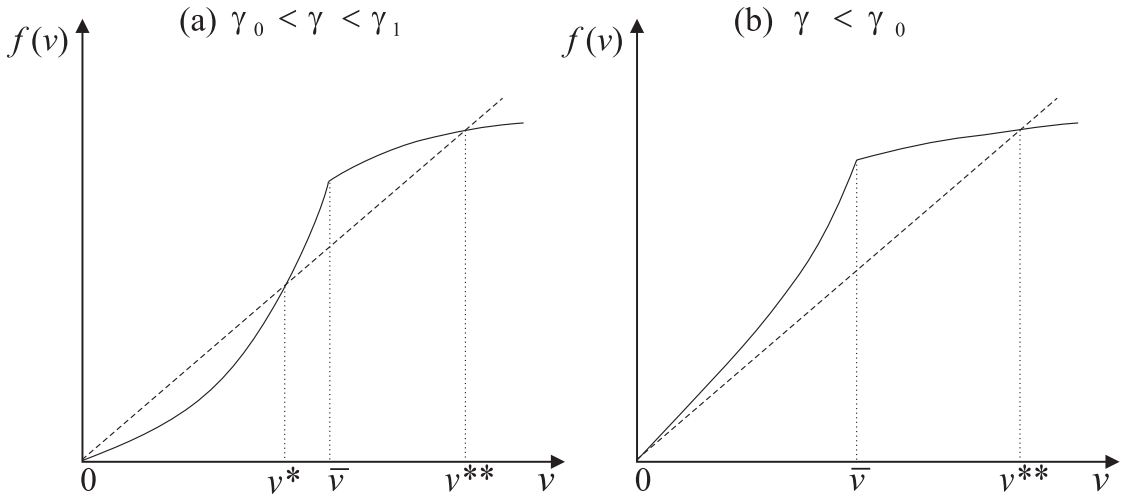

Figure 1: Steady states at $v = 0, v^*, v^{**}$.

(c) *For $\gamma \leq \gamma_0$, there exists a unique stationary equilibrium with unsecured credit and efficient capital allocation at reputation value $v^{**} \in (\overline{v}, v_{max})$.*

The explanation for equilibrium multiplicity is a *dynamic complementarity* between endogenous credit constraints (reputation values). Borrowers' expectations of future credit market conditions influence their incentives to default which in turn determine current credit constraints. If future constraints are tight, the payoff of a clean credit record is modest so that entrepreneurs value access to unsecured credit only little. In turn, current default–deterring credit limits must be small or even zero. Conversely, if entrepreneurs expect future credit markets to work well, a clean credit record has high value, and this relaxes current constraints.

Figure 2 shows how stationary debt–equity limits θ depend upon the fundamental parameter γ. For small idiosyncratic productivity fluctuations ($\gamma > \gamma_1$), unsecured credit is not available because entrepreneurs value participation in credit markets too little: Secured borrowing ($\theta = \theta^c$) is the unique stationary equilibrium outcome. For larger idiosyncratic shocks, unsecured credit is sustainable because exclusion from future credit is a sufficiently strong threat for borrowers. For intermediate values of γ, two further steady states emerge: one with an inefficient capital allocation at θ^*, and one with an efficient capital allocation at $\theta^{**} = \frac{1-\pi}{\pi}$. For large enough idiosyncratic shocks ($\gamma < \gamma_0$), the unique steady state with unsecured credit has an efficient capital allocation.

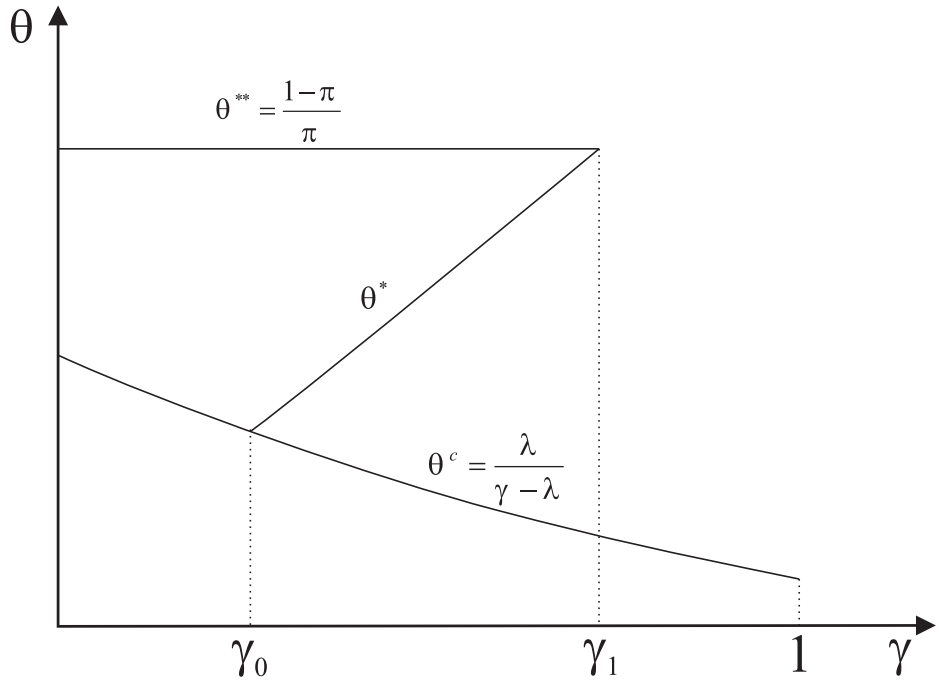

Figure 2: Steady-state debt-equity ratios θ for varying productivity differential $\gamma = a^u/a^p$.

But even when unsecured credit permits efficient allocations of capital, efficiency rests upon the confidence of market participants in future credit market conditions. When market participants expect credit constraints to tighten rapidly, the value of reputation shrinks over time which triggers a self–fulfilling collapse of the market for unsecured credit. For instance, if $\gamma < \gamma_0$, the steady state at v^{**} is determinate and the one at $v = 0$ is indeterminate (see Figure 1(b)). That is, there exists an infinity of non–stationary equilibria $v_t = f(v_{t+1}) \to 0$ where the value of reputation vanishes asymptotically.[13] If $\gamma \in (\gamma_0, \gamma_1)$, the two steady states at $v = 0$ and at v^{**} are determinate, but the one at v^* is indeterminate. That is, there is an infinity of non–stationary equilibria $v_t \to v^*$.

Indeterminacy in our model not only allows collapsing credit market bubbles, it also permits business cycles driven by self–fulfilling beliefs (sunspots). To see this, consider any sequence of random variables $\varepsilon_{t+1} \in (-v_t, v^{**} - v_t)$, $t \geq 1$, satisfying

[13]These equilibria are mathematically similar to hyperinflationary equilibria in overlapping–generation models of fiat money but do not require the existence of an unproductive financial asset.

$E(\varepsilon_{t+1}|v_t) = 0$, and define the stochastic process

$$v_{t+1} = f^{-1}(v_t + \varepsilon_{t+1}) \in (0, v^{**}) \ . \tag{13}$$

Then, the stochastic cycle (13) is a solution of equation (12). Sunspot fluctuations vanish asymptotically if $\gamma < \gamma_0$, but they give rise to permanent volatility around the indeterminate steady state v^* if $\gamma \in (\gamma_0, \gamma_1)$.

Proposition 4 *Suppose that $\gamma \in (\gamma_0, \gamma_1)$ as defined in Proposition 3. Then there exist sunspot cycles featuring permanent fluctuations in credit, output and total factor productivity.*

The dynamic complementarity between endogenous credit constraints not only gives rise to expectations-driven business cycles, it also generates an endogenous propagation mechanism of sunspot shocks. This follows because $f'(v^*)$ is positive: a one-time expectational shock in period t triggers a persistent adjustment dynamics of reputation values v_k (and thus of credit constraints, investment and output) in subsequent periods $k > t$. Intuitively, a self-fulfilling credit boom (slump) in period t can only emerge if the boom (slump) lasts for several periods. The next section demonstrates that sunspot shocks have the ability to account quantitatively for persistent dynamics of key macroeconomic variables with plausible relative volatilities.

Corollary 2 *A one-time sunspot shock $\varepsilon_t > 0$ ($\varepsilon_t < 0$) in period t induces a persistent positive (negative) response of firm credit and output.*

5 A quantitative illustration

We calibrate the model to the U.S. economy to study the quantitative features of endogenous cycles and to explore how the model economy responds to different shocks. We choose a quarterly period length and parameters such that the model's indeterminate steady state equilibrium matches suitable long-run properties of the U.S. economy. Note again that only the indeterminate steady state equilibrium allows for unsecured credit and inefficient capital allocations (Proposition 3). The other two determinate steady states of this model either feature efficient factor allocations or do not sustain unsecured credit. Hence their business-cycle properties

would either resemble those of a standard frictionless model or those of an economy with exogenous, collateral-based credit constraints.[14]

We set $g = 0.005$, $\delta = 0.015$ and $\alpha = 0.33$ to match plausible values of output growth, capital depreciation and capital income shares. We normalize average capital productivity in steady state to $a = 1$ and set $\beta = 0.987$ to match a steady state capital-output ratio of 10. We choose the remaining parameters π, λ and a_u to match the following three targets:[15] (1) Credit to non-financial firms is 0.5 of annual GDP; (2) a debt-equity ratio $\theta = 3$ of credit-constrained firms; (3) cross-firm dispersion of annual equity returns of 0.4.[16] Given that this model has a two-point distribution of firm productivity (and hence of debt-equity ratios), the choice of target (2) is somewhat arbitrary. As a robustness check, we also calibrate the model with $\theta = 2$ and report business-cycle statistics below.[17] Since only $\pi = 6.7\%$ of firms have positive investment rates in each quarter, this parameterization accounts for investment spikes similar to those documented by Cooper and Haltiwanger (2006). The calibrated model also yields an annual dispersion of employment growth of 0.51 which is in line with Davis, Haltiwanger, Jarmin, and Miranda (2006). We set the exclusion parameter $\psi = 0.025$ so that an average entrepreneur has difficulty obtaining unsecured credit for a period of 10 years after default.[18] In this model parameterization, 45% of all firm credit is secured and 55% is unsecured. Unsecured borrowing boosts allocative efficiency quite substantially: aggregate output would drop by 21% if only secured credit was available. Our parameter choices are summarized in Table 1.

[14] Different from our model, collateral-based credit constraints may also be forward-looking as in Kiyotaki and Moore (1997) if collateral prices are endogenous, which may contribute to the amplification and propagation of shocks. Cordoba and Ripoll (2004) and Kocherlakota (2000) argue, however, that it is difficult to generate quantitatively significant amplification.

[15] The normalization $a = a^u + \pi(1+\theta)(a^p - a^u) = 1$ then pins down parameter a^p.

[16] (1) Credit market liabilities of non-financial business are 0.52 of annual GDP (average over 1952-2011, Flow of Funds Accounts of the Federal Reserve Board, Z.1 Table L.101). (2) Debt-equity ratios below 3 are usually required to qualify for commercial loans; indeed, more than 80% of small businesses have debt-equity ratios below 3 (see Herranz, Krasa, and Villamil (2009), Figure 2). Note, however, that the average debt-equity ratio is much lower both in the model (0.2) and in the Flow of Funds Accounts for non-financial business (0.41, average for 1952-2011). On (3), see Figure 1 in Jiang (2008).

[17] Departing from Table 1, this requires $a^u = 0.94$, $a^p = 1.14$, $\pi = 0.1$ and $\lambda = 0.12$.

[18] For example, if a firm owner files for bankruptcy according to Chapter 7 of the U.S. Bankruptcy Code, bankruptcy remains on the credit record for a period of 10 years (see e.g. Chatterjee, Corbae, Nakajima, and Rios-Rull (2007)).

Table 1: Parameter choices.

Parameter	Value	Description	Target
g	0.005	Efficiency growth	Output growth
δ	0.015	Depreciation rate	Consumption of fixed capital
α	0.33	Production fct. elasticity	Capital income share
β	0.987	Discount factor	Capital-output ratio
π	0.067	Share of productive firms	Credit of non-financial firms
λ	0.48	Collateral share	Debt-equity ratio $\theta = 3$
a^u	0.95	Lowest productivity	Equity return dispersion
a^p	1.138	Highest productivity	Normalization $a = 1$
ψ	0.025	Exclusion parameter	10-year punishment

To explore expectations-driven business cycles, we feed this model with a sunspot shock process that generates the magnitude of output volatility in U.S. data. This counterfactual exercise allows us to understand how the economy would fluctuate if sunspot shocks were the only shocks affecting the business cycle. We then compare the cyclical response of output, firm credit and investment to those in the data. We choose a process of the form $v_{t+1} = f^{-1}(v_t + \varepsilon_{t+1})$ with sunspot shocks

$$\varepsilon_{t+1} = \begin{cases} -\sigma v_t & \text{with prob. } \zeta_t \ , \\ \sigma(v^{**} - v_t) & \text{with prob. } 1 - \zeta_t \ , \end{cases}$$

where probability $\zeta_t = (v^{**} - v_t)/v^{**}$ ensures that $E_t(\varepsilon_{t+1}) = 0$. This specification guarantees that reputation values v_t stay in the interal $(0, v^{**})$ in all periods. Parameter σ determines the magnitude of sunspot shocks which is the only free parameter in this two-point sunspot process. Note that we cannot allow for another parameter governing the persistence of sunspot shocks since $E_t(\varepsilon_{t+1}) = 0$ rules out that these shocks are (positively or negatively) autocorrelated. We set $\sigma = 0.023$ ($\sigma = 0.017$) if $\theta = 3$ ($\theta = 2$) which implies that the model's simulated output time series has the same volatility as U.S. GDP.

Table 2 presents U.S. business cycle statistics and their counterparts in the simulated sunspot cycle for both calibrations. Notably, the model matches the relative volatility of firm credit and investment quite well. More importantly, the model generates persistent dynamics of output, credit and investment which are in line with the data. This is despite the fact that the underlying shock process has no

persistence. Hence the model generates endogenous propagation of independent self-fulfilling expectation shocks. Lastly, firm credit and investment are strongly procyclical, although correlations with output are larger than in the data.

Table 2: Business cycle statistics with sunspot shocks

	Output	Firm Credit	Investment
U.S. data			
Standard deviation	0.026	0.056	0.103
Autocorrelation	0.938	0.985	0.896
Correlation with output	1	0.519	0.849
Model ($\theta = 3$)			
Standard deviation	0.026	0.066	0.127
Autocorrelation	0.895	0.900	0.877
Correlation with output	1	0.976	0.916
Model ($\theta = 2$)			
Standard deviation	0.026	0.064	0.134
Autocorrelation	0.892	0.898	0.872
Correlation with output	1	0.998	0.971

Notes: All variables are logged and HP filtered with parameter 10^5. U.S. data are for the period 1952Q1-2011Q4 and seasonally adjusted. Firm Credit includes all credit market instruments of non-financial business in the Flow of Funds Accounts of the Federal Reserve Board (series FL144104005 in Table L.101). GDP and investment are in chained 2005 dollars from NIPA. The model statistics are obtained from 1000 simulations of 240 periods.

Our result on the internal propagation of sunspot shocks is very different from the economy's response to various types of shocks to fundamentals. To see this, we study the model dynamics under three different hypotheses on the sources of business cycles that have been proposed in the literature.[19] First, we consider "financial shocks" which are modeled as shocks to the collateral share parameter λ; second, we explore the effects of "uncertainty shocks", where parameters a^u and a^p fluctuate such that mean productivity is constant; third, we allow for fluctuations in the

[19]Stochastic TFP (A_t) would be another obvious candidate. In our model, however, the dichotomy between credit markets (equation (12)) and output/capital (equation (11)) implies that fluctuations in A_t take no impact on credit markets and hence work similarly as in a frictionless model.

number of firms undergoing investment spikes, parameter π.[20] In any of these three cases, we suppose that the respective model parameter follows a simple symmetric i.i.d. process around their calibrated figures in Table 1. As in Table 2, we calibrate the standard deviation of the shocks so as to match output volatility in the data. By abstracting from any autocorrelation in the underlying shock process, we isolate the endogenous propagation of fundamental shocks and compare it to the one with uncorrelated sunspot shocks. Table 3 shows the results of this exercise. While all fundamental shocks roughly match the relative volatility of firm credit, there is no internal propagation. Furthermore, all fundamental shocks generate somewhat higher investment volatility.

Table 3: Business cycle statistics with fundamental shocks

	Output	Firm Credit	Investment
Financial shocks (λ)			
Standard deviation	0.026	0.067	0.145
Autocorrelation	-0.019	-0.017	-0.029
Uncertainty shocks (a^u/a^p)			
Standard deviation	0.026	0.068	0.145
Autocorrelation	-0.007	-0.005	-0.020
Shocks to investment spikes (π)			
Standard deviation	0.026	0.050	0.147
Autocorrelation	-0.021	-0.014	-0.032

Notes: All variables are logged and HP filtered with parameter 10^5. Model statistics are obtained from 1000 simulations of 240 periods.

The contrasting propagation patterns of sunspot shocks and fundamental shocks are also shown in the impulse response diagrams of Figure 3. The upper graphs show the responses of output and firm credit to an expectation shock in period 1. The shock generates strong propagation with a long-lasting hump-shaped pattern peaking after 30 quarters. The responses to a financial shock in period 1 are

[20]For example, Kiyotaki and Moore (2008) and Jermann and Quadrini (2012) study financial shocks as exogenous fluctuations of borrowing constraint parameters; Bloom, Floetotto, and Jaimovich (2009) and Arellano, Bai, and Kehoe (2011) propose uncertainty shocks; and Gourio and Kashyap (2007) argue that the extensive margin of investment largely accounts for aggregate investment dynamics.

shown in the lower graphs. A one-time increase of the collateral fraction λ by 5% generates an output increase by 7.6% in period 1 which does not propagate into subsequent periods. The absence of internal propagation of fundamental shocks is hardly surprising, given that economies with binding (exogenous) credit constraints have difficulty to generate propagation (see Cordoba and Ripoll (2004)). Also, some of the traditional fundamental propagation channels, such as endogenous wealth dynamics, adjustment costs or habit persistence, are absent from our model.

Why do sunspot shocks have a long-lasting impact whereas fundamental shocks do not? Intuitively, the forward-looking nature of reputation values necessitates that a self-fulfilling boom of unsecured credit can only be generated if the boom is long-lasting. If credit market participants would expect that the boom decays quickly, such expectations could not induce a substantial credit expansion in the first place. Put differently, the dynamic forward-looking complementarity between reputation values triggers the endogenous propagation of self-fulfilling credit booms. Fundamental shocks, on the other hand, cannot generate similar propagations. Although such shocks also induce endogenous long-lasting responses of reputation values (and thus of unsecured credit), these are quantitatively insignificant relative to the direct impact of the fundamental shock. This becomes evident from the bottom graphs in Figure 3 which show that the impact on output and firm credit after period 2 is almost negligible. One-time fundamental shocks can only generate long-lasting macroeconomic effects if they are correlated with expectational shocks, and, therefore, with the provision of credit.

6 Conclusions

Two apparent characteristics of the business cycle are the high autocorrelations of credit and output time series, and the strong cross-correlation between those two statistics. Understanding these correlations, without the help of persistent shocks to the productivity of financial intermediaries and final goods producers, has been a long-standing goal of macroeconomic research and the motivation for the seminal contributions mentioned in the first paragraph of the introduction to this paper. Is it possible that cycles in credit, TFP and output are not the work of persistent productivity shocks that afflict all sectors of the economy simultaneously? Could these cycles instead come from small and temporary shocks to anticipated credit

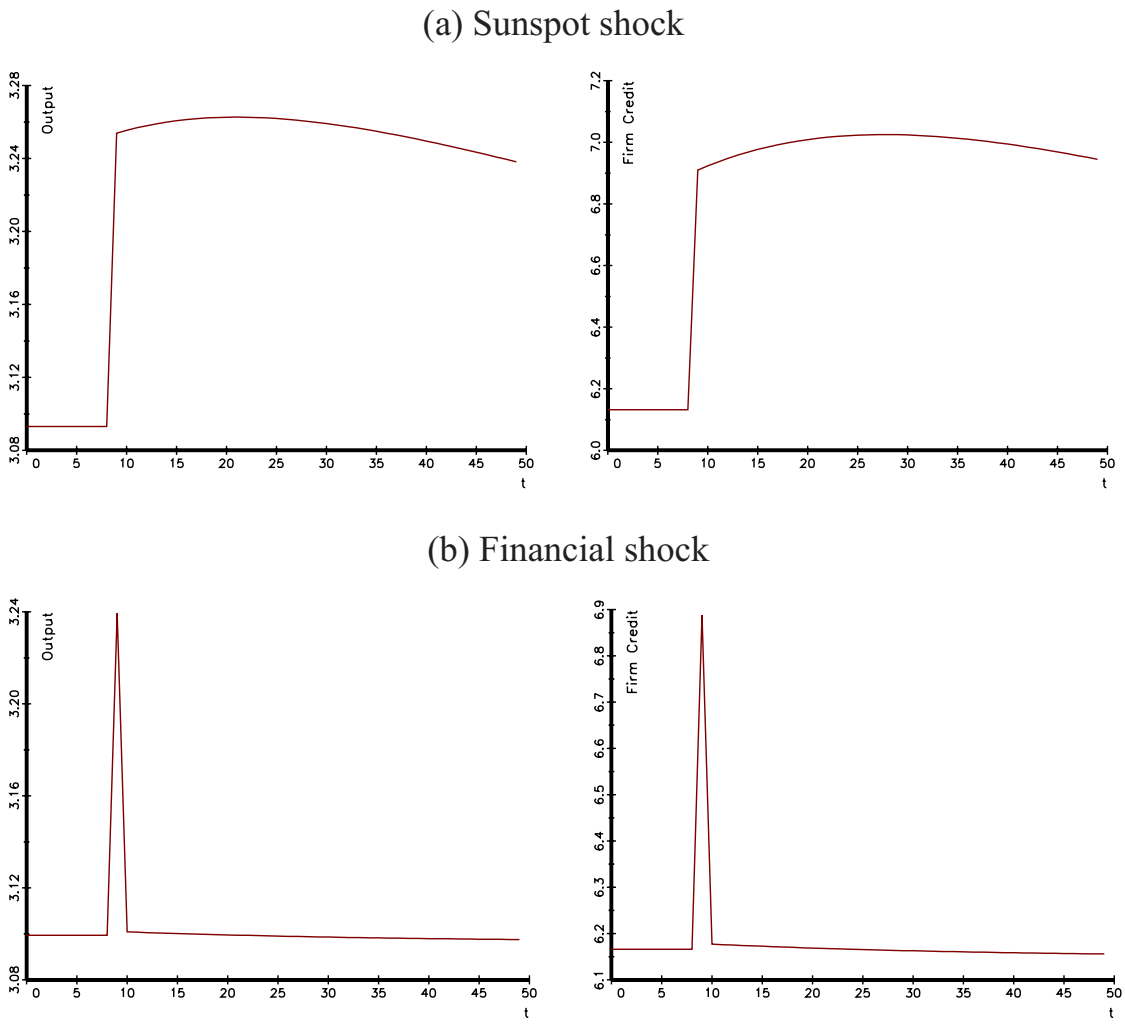

Figure 3: Impulse responses to a sunspot shock ($\varepsilon_{10} = 0.05$, $\varepsilon_t = 0$ for $t \neq 10$) and to a one-time financial shock ($\lambda_{10} = 0.504$, $\lambda_t = 0.48$ for $t \neq 10$).

conditions?

This paper gives an affirmative answer to both questions within an economy in which part of the credit firms require to finance investment is secured by collateral, and the remainder is based on reputation. Unsecured credit improves debt limits, facilitates capital reallocation and helps aggregate productivity, provided that borrowers expect plentiful unsecured credit in the future. Favorable expectations of future debt limits increase the value of remaining solvent and on good terms with one's lenders. Widespread doubts, on the other hand, about future credit will lead to long-lasting credit tightening with severe macroeconomic consequences.

It is this dynamic complementarity of current with future lending that connects macroeconomic performance over time and endows one-time expectational impulses with long lasting responses. A calibrated version of our economy, despite its apparent simplicity, matches well with observed features of the joint stochastic process governing U.S output, firm credit and investment.

Our economy shares with the literature on sunspots, bank panics and other multiple equilibrium models the drawback of arbitrary and exogenous equilibrium selection simply because we do not yet possess a convincing and well articulated theory of how economic actors focus on one outcome when many are available. One reasonable proposal, explored by Manuelli and Peck (1992) and Bacchetta, Tille, and van Wincoop (2010), is to think of expectation shocks as correlated with, and amplifiers of, small shocks to fundamentals. That idea seems to be worth another look.

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Appendix A: Autocorrelated productivity

To be written.

Appendix B: Proofs

Proof of Proposition 2: If borrowers were unconstrained in all periods, unproductive entrepreneurs lend out all their capital to productive entrepreneurs who borrow $(1-\pi)K_t$ in the aggregate, and the interest rate equals the capital return of productive entrepreneurs, $R_t = a^p R_t^*$. It follows that there are no gains from leverage so that $\tilde{R}_t = \tilde{R}_t^c = R_t$ for all $t \geq 0$, and the only solution to equation (10) is $v_t = 0$ for all t. But then it follows from equation (9) that debt-equity constraints are $\theta_t = \theta_t^c = \lambda/(1-\lambda)$, and the restriction on collateral ($\lambda < 1-\pi$) implies

$$\theta_t \pi K_t < (1-\pi)K_t \ .$$

Thus, aggregate borrowing exceeds the aggregate credit limit, a contradiction. □

Proof of Corollary 1: In a balanced–growth path, $1+g = K_{t+1}/K_t = \beta a R^*$, where $a = a^u + (a^p - a^u)\min(1, \pi(1+\theta))$ is average capital productivity. From Proposition 2, any steady state has binding credit-constraints, so that $R = \rho a^p R^* < a^p R^*$. Then either $\rho = \gamma$ implies $R = a^u R^* < aR^*$, or $\rho > \gamma$ and (5) implies $\pi(1+\theta) = 1$, so that $a = a^p$ and again $R < aR^*$. In any case, $R < (1+g)/\beta$ follows. □

Proof of Proposition 3: Because of $f(v_{max}) < v_{max}$ and continuity, a solution $f(v) = v \in (\overline{v}, v_{max})$ exists iff $f(\overline{v}) > \overline{v}$. This condition is

$$[1 - \gamma(1-\pi)]^{1+\Phi} > \left[\frac{\pi\gamma}{\gamma - \lambda}\right]^{\Phi} (1-\lambda)^{1+\Phi} , \tag{14}$$

with $\Phi = \beta\pi/(1-\beta(1-\psi))$. Both sides in (14) are decreasing in γ and LHS<RHS at $\gamma = 1$ (because of $\lambda < 1 - \pi$). Further, at $\gamma = \lambda/(1-\pi)$, LHS=RHS and the LHS has a bigger slope than the RHS (because of $\lambda < \frac{\beta(1-\pi)}{1+\beta\psi}$). Therefore there exists another intersection $\gamma_1 \in (\frac{\lambda}{1-\pi}, 1)$, such that LHS>RHS for all $\gamma \in (\frac{\lambda}{1-\pi}, \gamma_1)$. It follows that the steady state $v^{**} \in (\overline{v}, v_{max})$ exists if $\gamma < \gamma_1$.

Since f is strictly convex in $v \in (0, \overline{v})$, a steady state $v^* \in (0, \overline{v})$ exists if $\gamma < \gamma_1$ (implying $f(\overline{v}) > \overline{v}$) and if $f'(0) < 1$. The latter condition is equivalent to $\gamma > \gamma_0 \equiv \frac{\Phi+\lambda}{\Phi+1}$. Moreover, $\lambda < \frac{\beta(1-\pi)}{1+\beta\psi}$ also implies that $\gamma_0 > \frac{\lambda}{1-\pi}$ (so that credit constraints are indeed binding). This completes the proof. □